# Fiscal sustainability in Japan*

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**Abstract**

We investigate fiscal sustainability of Japan by providing a dynamic stochastic general equilibrium (DSGE) model that features low interest rates of government bonds relative to the economic growth rate. We evaluate sustainability by testing whether the expected debt-to-GDP ratio stabilizes or increases without bound. The simulated debt-to-GDP ratio depends on the intermediation cost, the elasticity of intertemporal substitution, the projected growth rate, and on the specified fiscal policy rule. If the fiscal policy rule estimated over the past 30 years goes over in the future, the debt-to-GDP ratio will increase without bound, and in this sense the fiscal policy is not sustainable. We investigate alternative fiscal policy rules to avoid possible fiscal insolvency.

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## 1. Introduction

Whether government debt is sustainable in Japan has become a great concern. Japan's government debt outstanding is close to the double of annual GDP, which is the highest among the developed countries and in its own post-WWII history. Though the Japanese government (Cabinet Decision, 2010) declared its target at turning from primary deficit to primary surplus by 2020 and lowering the debt-to-GDP ratio from 2021, the weak economic recovery and unstable political situations are undermining its feasibility and credibility. We investigate the sustainability of Japanese government debt under the fiscal policy observed in the past thirty years and some alternative policy rules.

Investigating fiscal sustainability, however, entails facing a puzzling fact. Interest rates on government bonds have remained quite low relative to the economic growth rate in Japan. Figure 1 illustrates the time series of the Financial bill rate, the interest rate of the long-term bond, and the economic growth rate for the period of 1981–2009. All figures are measured in real terms in terms of GDP deflator. The averages of the long-term bond and the Financial bill rates are 3.6 percent and 1.9 percent, respectively, while the average of the growth rate is 2.2 percent.

With the annual discount factor that is extensively used in the business cycle literature (namely, $1/1.03$, e.g., Christiano and Eichenbaum, 1992 and Christiano, et al, 2005), both the exogenous growth model and the endogenous growth model with log-utility predict that the interest rate should be 3 percent above the growth rate. Understanding a “low” interest rate is crucial to investigating fiscal sustainability.[1]

To account for low interest rates of government bonds, we provide a dynamic stochastic general equilibrium (DSGE) model incorporating financial friction and the heterogeneity in the access to production among agents (e.g., Woodford, 1990 and Bohn, 1999). The introduction of the intermediation cost gives rise to declines in both the economic growth rate and the interest rate of government bond, allowing the model to mimic the actual data with the gap increasing or decreasing depending on the elasticity of substitution on consumption. If the elasticity is nearly less than 2, the intermediation cost decreases the gap, and hence improve sustainability.

We evaluate sustainability by testing whether the debt-to-GDP ratio stabilizes or increases without bound. The simulated debt-to-GDP ratio depends on the intermediation cost, the elasticity of intertemporal substitution of consumption, the projected growth rate, and the specified fiscal policy rule. Of particular importance is the elasticity of intertemporal substitution. Our estimate is within the range over which the high intermediation cost leads to a decline in the gap between the growth and interest rates, contributing to the improvement of fiscal sustainability. At least as for the Japanese case, our approach that explains low interest rates imposes a looser condition on sustainability than the standard model otherwise.

The specification of the fiscal rule is crucial to evaluate fiscal sustainability. If the fiscal rule that has been adopted over the past thirty years goes over in the future, the

[1] This tendency is common to many countries. The average realized real rates of return on government bonds in major OECD countries over the past 30 years have been smaller than the real growth rate (e.g., Blanchard and Weil, 2001).

expected debt-to-GDP ratio would reach 11.5 in 100 years and afterwards would continue to grow unboundedly. The probability that the debt-to-GDP ratio will diverge is greater than 50 percent in 20 years and later, and we have to judge that the Japanese fiscal policy is not sustainable. If the fiscal rule incorporates Bohn's idea that a rational government should increase the primary surplus when the debt-to-GDP ratio is high, sustainability improves.

We do not rely on the risk-premium approach to explain low interest rates for two reasons (e.g., Mehra and Prescott, 1985, and Weil, 1989).[2] First, this approach explains only the low interest rate of the safe bond but does not explain low rates of the government bonds as a whole. Secondly, as the literature on the "risk-free rate puzzle" (e.g., Weil, 1989) points out, classes of simple utility functions do not explain the low interest rate within admissible parameter values.

This paper contributes to the literature on methodology to test fiscal sustainability. We do not use the approach of checking the intertemporal government budget constraint in two respects. First, the latter approach constitutes only a necessary condition for sustainability. Secondly, in the presence of intermediation costs, among various menus of government bonds, any interest rate and/or their combination is not appropriate for discounting the future correctly.

[2] Abel et al. (1989) and Bohn (1995) provide stochastic growth models in which the risk premium drives down the safe interest rate, often below the economic growth rate. Their argument demonstrates that the no-Ponzi condition for the intertemporal government budget constraint holds, even if the safe interest rate is below the growth rate.

Hamilton and Flavin (1986), Ahmed and Rogers (1995), and others use the approach of investigating the intertemporal government budget constraint.[3] Bohn (1995) criticized this approach for the reason that safe government bonds do not reflect correct discounting. Observing the lower interest rates of the US government bond relative to the growth rates in many periods, Bohn (1998) proposed a simple test to check whether the debt-to-GDP ratio displays a mean-reversion property.[4] Ball et al. (1998), in their famous paper entitled "Deficit Gamble," projected future growth rates and interest rates from past data and calculated the probability under which the debt-to-GDP ratio would enter a dangerous zone. Our approach is similar to the latter approach. In our model, when there is a significant intermediation cost, the government can run the Ponzi strategy, but even then, if the debt-to-GDP ratio is constant, the fiscal policy is sustainable.

Recently, some literature has studied fiscal sustainability by applying a dynamic stochastic general equilibrium (DSGE) model. Mendoza and Oviedo (2004, 2006) develop small open economies to investigate how macroeconomic shocks affect government finances and estimate the amount of sustainable public debt in emerging market economies. Arellano (2008) also develops a small open-economy model to study sovereign default risk and its interaction with output and foreign debt. Sakuragawa and

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[3] Other works include Wilcox (1989), Trehan and Walsh (1991), Hakkio and Rush (1991),, Uctum and Wickens (2000), and Polito and Wickens (2005).

[4] Bohn (2005) applies his test to the historical data of the US and finds evidence supporting fiscal sustainability (see also Greiner and Kauermann, 2007). Mendoza and Ostry (2008) apply Bohn's test to industrial and emerging countries and finds evidence of fiscal solvency in both types of countries. Galí and Perotti (2003) and Wierts (2007), among others, apply Bohn's test to European countries.

Hosono (2009) develop a closed economy model of an exchange economy to test fiscal sustainability of the Japanese economy. In the present paper, we extend their model by incorporating the AK-type production technology, which enables us to investigate the effect of the financial intermediation cost on the growth-interest rate relationship. We also expand the range of fiscal policy rules that we simulate, based on the observed rule over the last thirty years. Sakuragawa et al (2010) use a closed economy model of a production economy to test fiscal sustainability of the US.

This paper is also related to the theoretical literature that combines financial frictions with the heterogeneity in the access to production among agents to have implications for the lower interest rate than the growth rate. The literature includes Woodford (1990), Bohn (1999), Kiyotaki and Moore (2008), Hellwig and Lorenzoni (2009) and Kocherlakota (2009).

This paper is organized as follows. In Section 2, we outline the model. In Section 3, we develop the theoretical analysis. In Section 4, we describe the simulation procedure. In Section 5, we investigate the sustainability of the Japanese public debt. Section 6 concludes.

## 2. Model

Consider an economy made up of two types of agents that live infinitely, with the number of each normalized to be unity, and the third type of agents that live for two periods and act as intermediaries. We consider heterogeneous agents and financial friction in order to provide implications for low interest rates.[5]

Type E agents have access in all *even* periods to an AK production technology that transforms $K_t$ units of the final good into random $(1+x_{t+1})K_t$ units after one period, while type O agents have access in all *odd* periods. Two reasons motivate the introduction of the AK model. First, fiscal sustainability is a long-run problem. Second, the AK model enables one to have the positive link between interest and growth rates that is observed in the time series. The rate of return on capital, $x_{t+1}$, is a random variable that follows a Markov process and takes values in a set, $X_t$. The history of the economy up to time $t$ is denoted by $h_t = (x_t, x_{t-1}, \ldots)$, which takes values in a set $H_t$. Denote the probability of a variable, $x_{t+1}$, given a history $h_t$, by $\pi(x\,|h_t)$.

To simplify the notation, let there be one representative agent of each type so that the individual income $(1+x_{t+1})K_t$ denotes the aggregate income. There is no population growth. Both types have identical preferences over consumption and maximize $E_0\sum_{t=0}^{\infty}\beta^t\frac{C_t^{1-\alpha}}{1-\alpha}$, where $\alpha$ is the inverse of the elasticity of substitution of consumptions across periods, $\beta$ (0< $\beta$ <1) is the discount factor, and $E_0$ is the expectation operator. We impose the relevant condition on bounded utility by $E_t\{\beta(1+g_{t+1})^{1-\alpha}\}<1$, where $g_t$ is the growth rate of the aggregate income argued below. The government spending,

[5] Aiyagari and Gertler (1991) and Heaton and Lucas (1996) construct models with intermediation costs and heterogeneous agents.

$G_t$, is a constant share of GDP to meet $G_t = zY_t$. The government finances its spending by imposing lump-sum taxes and by issuing public debt.

At each period, finite $N$ agents who act as intermediaries are born and live for two periods. They are endowed with a specific skill of intermediating finance between private agents, and maximize the second-period consumption less the amount of effort exerted by them.

We introduce financial friction by supposing that these agents have to bear a proportional intermediation cost $\kappa$ per unit of funds. The cost is measured in terms of the loss of effort. One may interpret the intermediation costas a cost of monitoring or identifying a borrower, or of verifying credit.

## 3. Theoretical Analysis

The intermediary issues securities that request the rate of repayment $r_t^b$ to firms and guarantee the rate of return $r_t$ to investors. In a world of competitive intermediation, intermediaries finally have to earn zero profit to satisfy

(1) $$(1+r_t)(1+\kappa) = 1 + r_t^b$$

for any $x_t$. Note that both assets are risky in the sense that the rate of return depends on the productivity. We take an approach of the incomplete bond market where the government can issue only one-period bonds and private agents cannot insure away the income uncertainty.[6]

At the beginning of an even period $t$, type E agents face a shock $x_t$, receive capital income $(1+x_t)K_{t-1}$, repay $(1+r_t^b)B_{t-1}$, consume $C_t$, pay taxes $T_t^E$, and invest the remaining in the private security $W_t$ and the public bond $D_t$. They maximize the following value function:

(2) $$V(B_{t-1},K_{t-1},x_t) = \max_{C_t,W_t,D_t} \frac{C_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1}\in X_{t+1}} \pi(x_{t+1}|h_t)\,\tilde{V}(W_t,D_t,x_{t+1})$$

subject to the budget constraint

(3) $$(1+x_t)K_{t-1} - (1+r_t^b)B_{t-1} = C_t + W_t + D_t + T_t^E\,.$$

On the other hand, at period $t$, type O receives interest incomes from the private security $(1+r_t)W_t$ and the public bond $(1+R_t)D_{t-1}$, and transfers $\Pi_t$ from intermediaries. They consume $\tilde{C}_t$, pay taxes $T_t^O$, and invest the remaining in capital to produce in the odd period. They maximize the following value function:

(4) $$\tilde{V}(W_{t-1},D_{t-1},x_t) = \max_{\tilde{c}_t,K_t,B_t} \frac{\tilde{C}_t^{1-\alpha}}{1-\alpha} + \beta \sum_{x_{t+1}\in X_{t+1}} \pi(x_{t+1}|h_t)\; V(B_t,K_t,x_{t+1})$$

subject to

(5) $$\Pi_t + (1+r_t)W_{t-1} + (1+R_t)D_{t-1} + B_t = \tilde{C}_t + K_t + T_t^O,$$

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6 Several studies cast doubts on the presence of complete bond markets. For example, Marcet and Scott (2009) find the persistency of the data for the US government debt, which is supportive of incomplete markets but is inconsistent with complete markets

where $R_t$ is the interest rate on the government bond, and $\Pi_t (\equiv \kappa(1+r_t)W_{t-1})$ is the intermediary's profit that is transferred to them. The intermediary is compensated for the loss of effort by income, but the income accruing to the intermediary is transferred to households in a lump-sum fashion.

Assume that the equilibrium has an interior solution. Equilibrium conditions on $K_{t+1}$, $W_{t+1}$, $B_{t+1}$, and $D_{t+1}$, together with envelope conditions, lead to

(6) $$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1}|h_t)(1 + x_{t+1}) \frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1}|h_t)\}^{\alpha}}$$

(7) $$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1}|h_t)\{1 + r(x_{t+1}|h_t)\} \frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1}|h_t)\}^{\alpha}}$$

(8) $$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1}|h_t)\{1 + r^b(x_{t+1}|h_t)\} \frac{\{\tilde{C}(h_t)\}^{\alpha}}{\{C(x_{t+1}|h_t)\}^{\alpha}},$$

and

(9) $$1 = \beta \sum_{x_{t+1} \in X_{t+1}} \pi(x_{t+1}|h_t)\{1 + R(x_{t+1}|h_t)\} \frac{\{C(h_t)\}^{\alpha}}{\{\tilde{C}(x_{t+1}|h_t)\}^{\alpha}}.$$

The market clearing in the good market is expressed as

(10) $$C_t + \tilde{C}_t + K_t + zY_t = (1 + x_t)K_{t-1}.$$

The market clearing in the credit market is expressed as

(11) $$W_t = B_t.$$

Finally, the government's budget constraint is given by

(12) $$D_t = (1 + R_t)D_{t-1} + G_t - T_t^E - T_t^O.$$

As Barro (1979), Kremers (1989), and Bohn(1991) have argued, we impose an additional feasibility constraint restricting the government's taxable income to be limited to some fraction of the aggregate income. We simply call a fiscal policy *feasible* if the tax revenue does not exceed a fraction $\tau$ of GDP, that is, (FC) $$T_t^E + T_t^O \le \tau Y_t$$

at any time and at any state $s$.

The competitive equilibrium is defined as a sequence of nine variables $\{\frac{C_{t+1}}{C_t}, \frac{\tilde{C}_{t+1}}{\tilde{C}_{t+1}}, \frac{K_{t+1}}{K_t}, \frac{W_{t+1}}{W_t}, \frac{B_{t+1}}{B_t}, \frac{D_{t+1}}{D_t}, r_t, r_t^b, R_t\}_{t=0}^{\infty}$, satisfying nine equations (1), (3),(6)–(12), (FC), and relevant non-Ponzi-game conditions, given the sequence of random variables $\{x_t\}_{t=0}^{\infty}$ and the sequence of the policy rule $\{T_t^E, T_t^O, D_t\}_{t=0}^{\infty}$.

The fact that two-period-lived intermediaries have no intertemporal consideration simplifies the link among several interest rates. As for the link between $x_t$ and $r_t^b$, competitive intermediation should lead to $x_t = r^b(x_t)$ for any $x_t$. Jointly with (1), the investors' rate of return $r_t$ should also be dependent on $x_t$ to satisfy $r_t = (1 + r^b(x_t))/(1+\kappa) - 1 \equiv r(x_t)$. Equations (6) and (8) imply that the private security issued by the intermediary and the government bond are perfect substitutes for investors so that, without loss of generality, we may set $R(x_t) = r(x_t)$ for any $x_t$. We have $R(x_t) < r^b(x_t)$; the government can borrow at a lower rate than private agents. The reason behind this finding is that loans to the government can be monitored with no cost, while loans to private agents need intermediation cost.

We use these features on several interest rates to argue on the non-Ponzi-game condition relevant to the government bond. In private lending/borrowing, any individual agent is willing to choose to be on the *borrowing* side of a rational Ponzi game to satisfy

$E_t \lim_{s\to\infty} \frac{B_{t+s}}{(1+r^b_{t+1})(1+r_{t+2})(1+r^b_{t+3})...} \geq 0$, while on the other hand, the borrowing of the individual is constrained by the behavior of others who find it optimal not to be on the *lending* side of a Ponzi scheme. The constraint is represented by $E_t \lim_{s\to\infty} \frac{B_{t+s}}{(1+r^b_{t+1})(1+r_{t+2})(1+r^b_{t+3})...} \leq 0$.[7] These two considerations formalize the non-Ponzi-game condition for private lending/borrowing; $E_t \lim_{s\to\infty} \frac{B_{t+s}}{(1+r^b_{t+1})(1+r_{t+2})(1+r^b_{t+3})...} = 0$ (e.g., O'Connell and Zeldes, 1988).

In government lending, the government is not any optimizer so that the first condition is unnecessary (see Bohn 1995). On the other hand, the borrowing of the government is constrained by the behavior of agents who find it optimal not to be on the lending side of a Ponzi scheme. The constraint is $E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+r_{t+1})(1+r^b_{t+2})(1+r_{t+3})...} \leq 0$. Ignoring the possibility of individual's borrowing from the government, it becomes

(NPG1) $$E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+\kappa)^{s/2}(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...} = 0,$$

where we use (1) and $r_t = R^b_t$. The condition (NPG1) establishes the non-Ponzi-game condition for the government bond. On the other hand, the discounted value of debt that the government can earn by a Ponzi strategy, $E_t \lim_{s\to\infty} \frac{D_{t+s}}{(1+R_{t+1})(1+R_{t+2})...(1+R_{t+s})...}$, may be positive even under (NPG1) when $\kappa > 0$. Interestingly, (NPG1) does not rule out the government' Ponzi game *completely* because agents discount the future at the higher rate than the government does.

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[7] Agents alternate between a lender and a borrower every other period and discounts the future at rate $\sqrt{(1+r_t)(1+r^b_{t+1})} - 1$ and $\sqrt{(1+r^b_t)(1+r_{t+1})} - 1$.

The condition (NPG1) is necessary but not sufficient to establish a condition for sustainability. It restricts the debt to grow no faster than the rate of discount (multiplied by the term $(1+\kappa)^{1/2}$), but when the interest rate is greater than the economic growth rate, it permits the debt to grow faster than the economy as whole. An unbounded debt-output ratio cannot be ruled out only by (NPG1) (e.g., McCallum, 1984).

An additional constraint of feasibility, (FC), is necessary. Accordingly, it makes sense to define fiscal sustainability more narrowly. A fiscal policy rule is defined to be sustainable if the following two conditions are satisfied. First, (NPG1) is to be satisfied. Second, the government's budget is to be feasible, i.e., (FC) is satisfied at any time and at any state. The following is established.

**Proposition:** If fiscal policy is sustainable, then $E_t \dfrac{D_{t+s}}{Y_{t+s}}$ is bounded above.

*Proof.* See Appendix A.

The intuition behind the proof is as follows. Suppose that the debt-to-GDP ratio grows unboundedly. The government will either let debt grow further without increasing taxes, or constrain debt by increasing taxes. In the former case, debt will eventually exceed the amount that the private agents are willing to lend to the government, violating the condition (NPG1). In the latter case, tax revenues required will exceed the limit of taxability, violating the condition (FC). Satisfying both (NPG1) and (FC) requires the debt-to-GDP ratio to be bounded above.

An example of the deterministic economy shows that the sustainable fiscal policy requires $\dfrac{D_t}{Y_t} < \dfrac{(\tau - z)(1+g)}{R-g}$. Suppose, for example, that $R = 0.02$, $z = 0.1$, $\tau = 0.3$, and $g = 0.01$, then the upper bound becomes, $D_t / Y_t < 20.2$ which is extremely high so that the latter condition seems virtually not restrictive if the debt-to-GDP ratio is stable. Therefore, we may safely say that if the debt-to-income ratio does not grow unboundedly but converges to some level, the fiscal policy is sustainable.

Let $C_t / \tilde{C}_t \equiv \theta_t$ denote the consumption ratio between two different types of agents. Limiting focus on an economy with $\theta_t$ being constant through time, we have the consumption growth rate as

(13) $$\frac{C(x_{t+1}|h_t)}{C(h_t)} = \frac{\tilde{C}(x_{t+1}|h_t)}{\tilde{C}(h_t)} \equiv 1 + g(x_{t+1}|h_t).$$

We use (13) to rewrite (7) as

(14) $$1 = \beta \sum_{X_t} \pi(x_{t+1}|h_t)\{1 + g(x_{t+1}|h_t)\}^{-\alpha} \theta^{\alpha} \{1 + r(x_{t+1})\},$$

which embodies the relationship between the growth and interest rates. On the other hand, (8), (9), (13), and (1) jointly imply $\theta = (1+\kappa)^{1/2\alpha}$. In the presence of an intermediation cost, agents consume more when they receive income and consume less when they do not. On the other hand, plugging (1) and $\theta = (1+\kappa)^{1/2\alpha}$ into (14), we have the following:

(15) $$1 = \beta \sum_{X_t} \pi(x_{t+1}|h_t)\{1 + g(x_{t+1}|h_t)\}^{-\alpha} (1 + x_{t+1})(1+\kappa)^{-1/2}.$$

Equations (14) and (15) fully determine the sequence of the growth and interest rates as the stochastic variable $x_t$ evolves.

We explicitly solve (14) and (15). We denote the transition probability as $\pi(x_{t+1} = \lambda_j | x_t = \lambda_i) = \phi_{ij}$ in the set $X = \{x_1, ..., x_n\}$. We denote $g(x_{t+1} = x_j | h_t) = g_j$, which implies that the growth rate of consumption depends only on the current rate of return on capital. This arises from the fact that the production technology is the "AK" type and the fact that the proportion of government expenditure in output is constant. Accordingly, we rewrite (15) as

$$1 = \beta \sum_{j=1}^{n} \phi_{ij} (1+g_j)^{-\alpha} (1+x_j)(1+\kappa)^{-1/2} \quad \text{for} \quad i = 1,\dots,n \,. \tag{16}$$

We rewrite (14) similarly as

$$1 = \beta \sum_{j=1}^{n} \phi_{ij} (1+g_j)^{-\alpha} \theta^{\alpha} (1+r_j) \quad \text{for} \quad i = 1,\dots,n \,. \tag{17}$$

Equations (16) and (17) constitute $2n$ equations for solving $n$ growth rates and $n$ interest rates. We show the case of $n=2$, with $X = \{x_1, x_2\}$. We solve four variables $\{g_1, g_2, r_1, r_2\}$ from the following four equations:

$$\begin{bmatrix} (1+g_1)^{-\alpha}(1+x_1) \\ (1+g_2)^{-\alpha}(1+x_2) \end{bmatrix} = \beta^{-1}(1+\kappa)^{\frac{1}{2}} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \tag{18}$$

and

$$\begin{bmatrix} (1+r_1)(1+g_1)^{-\alpha} \\ (1+r_2)(1+g_2)^{-\alpha} \end{bmatrix} = \beta^{-1}(1+\kappa)^{-\frac{1}{2}} \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix}^{-1} \begin{bmatrix} 1 \\ 1 \end{bmatrix}. \tag{19}$$

Appendix B provides the procedure for solving the general case.

It is interesting to investigate how a change in the intermediation cost $\kappa$ influences $g_{t+1}$ and $r_{t+1}$, and their relationship. Equation (18) implies that a one percent increase in $\kappa$ leads to a decline of $g_{t+1}$ by approximately $1/(2\alpha)$ percent for each state. On the other hand, (19), together with (18), implies that a one percent increase in $\kappa$ leads to a decline of $r_t$ by one percent for each state. If $\alpha$ is sufficiently small (less than 0.5), or equivalently, the elasticity of substitution of consumptions is sufficiently large, the growth rate declines more sharply than the interest rate as $\kappa$ goes up. Figure 2A illustrates the case for $\alpha = 0.4$.[8] By contrast, if $\alpha$ is sufficiently large, the growth rate declines less sharply than the interest rate. Figure 2B illustrates the case for $\alpha = 1$ (log-utility). Whether higher intermediation costs make fiscal sustainability difficult or not depends on the elasticity of substitution of consumption.

## 4. Calibration

In this section and the next, we simulate the model to investigate the fiscal sustainability of Japan. Our methodology is to update the future fiscal variables by introducing a specified fiscal rule into the developed DSGE model and to simulate the debt-to-GDP ratio. The driving force of the growing economy is the rate of return on capital, which determines the growth rate and the interest rate. We specify the stochastic process for the logarithm of the gross rate of return on capital by discretizing a simple AR(1) process with nine states $(n = 9)$. The AR(1) process is described, with the serial correlation coefficient $\rho$ and the average $\log(1 + x^a)$, as

(20) $$\log(1 + x_{t+1}) = \rho \log(1 + x_t) + (1 - \rho)\log(1 + x^a) + e_{t+1},$$

where $e_{t+1}$ for all *t* are random shocks that are independent and identically distributed as a normal distribution with standard deviation $\sigma_e$. Once three parameters, $x^a$, $\rho$, and $\sigma_e$, have been set, following the method developed by Tauchen and Hussey (1991), we construct the nine states $\{x_1, \ldots, x_9\}$ and the transition probability $\phi_{ij}$ $(i, j = 1, \ldots 9)$.

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[8] In Figure 2, we set $x^a = 0.033$. See Table 2 for other parameter values.

We use (16) and (17) to solve for 18 variables $(g_j, r_j)$ $(j=1,...9)$, given the specified $(x_j, \phi_{ij})$ (see Sakuragawa, Hosono and Sano, 2010 for details). We use (20) to obtain the stochastic process for the GDP growth rate as a discretized version of an AR(1) process:

$$\log(1+g_{t+1}) = \rho\log(1+g_t) + (1-\rho)\log(1+g) + \varepsilon_{t+1}, \tag{21}$$

where $\varepsilon_{t+1}(\equiv e_{t+1}/\alpha)$ is a random shock with the standard deviation of $\sigma_\varepsilon(\equiv \sigma_e/\alpha)$ and $g$ is the steady-state growth rate.

The debt-to-GDP ratio evolves from (12) as

$$d_{t+1} = \frac{(1+R_{t+1})}{(1+g_{t+1})} d_t - s_{t+1}, \tag{22}$$

where $d_t$ and $s_t$ are debt and primary surplus divided by GDP, respectively. These two equations, together with a fiscal policy rule that determines $s_{t+1}$, provide the full system. Before going on to the fiscal policy rule, we choose parameter values. Data used to set parameters is described in Appendix C.

First, we choose the preference parameters, $\beta$ and $\alpha$. We set the annual discount factor $\beta$ to 1/1.02=0.980. The inverse of the elasticity of intertemporal substitution, $\alpha$, plays a central role in relating the interest rate to the growth rate, as captured by (19). To set $\alpha$, we note from (19) that

$$\log(1+R_j) = \alpha\log(1+g_j) + \gamma_j, \tag{23}$$

where $\gamma_j$ is the logarithm of the *j*th element of the right-hand side in (19). We regressed the nominal government bond yield on the nominal GDP growth rate using

OLS. The sample used for the estimation below covers the period of 1981-2009 except for otherwise mentioned. The estimation result is

(24) $$R_t = 0.023 + 0.565 g_t, \quad Adj.R^2 = 0.651,$$

$$(0.003) \quad (0.077)$$

where the numbers in parentheses are standard errors. Following (23), we set $\alpha$ at 0.565, implying that the elasticity of intertemporal substitution is set at 1.770. It is noteworthy that this figure is in the region of parameters in which a rise in the intermediation cost can improve fiscal sustainability.

Next, we specify the technology parameters, $\rho$ and $\sigma_e$. We apply (21) to conduct the OLS estimation of the AR(1) process of the GDP growth[9], obtaining

(25) $$g_{t+1} = 0.002 + 0.780 g_t, \quad Adj.R^2 = 0.4130.$$

$$(0.006) \quad (0.175)$$

We set $\rho$ at 0.780 and $\sigma_\varepsilon$ at 0.01996, where the latter is the root mean squared error of the regression. We use the chosen $\sigma_\varepsilon$ and $\alpha$ to set $\sigma_e$ at 0.01128. We set the average return on capital, $x^a$, at a value that yields 1 percent average GDP growth rate given the chosen values of $\beta$, $\alpha$, and $\kappa$ in (16). We find that $x^a$=0.033. The average GDP growth rate over the whole sample period of 1981-2009 is 2.2 percent, but this figure seems to be too high for predicting the future growth rate, especially when we consider the rapid population aging in Japan. We choose the average growth rate to

[9] We also estimated the AR(2) process and found that the two-year lagged growth rate is not significant and the adjusted R2 deteriorates as compared with (25).

be 1 percent considering that the average GDP growth rate over the period of 1990-2009 is 1.1 percent.[10]

Third, we set the financial intermediation cost, $\kappa$, at 0.015, the average of net interest margins between the bank loans and the bank deposits over the period of 2000–2009. We choose the average over the last decade because, as Figure 3 shows, the net interest margins tended to decline over the last two decades especially at a high rate in the 1990s.

We complete the model by specifying the government's fiscal policy rule. The fiscal rule is interpreted as a consequence of the conflict of interests among many pressure groups and so changes little unless the political situation changes drastically. Based on this idea, we specify the fiscal rule by the regression. We regress the primary balance as a proportion of GDP on the real GDP growth rate and the one-period lagged primary balance. The GDP growth rate is expected to capture the business cycle effects. When the economic boom comes, an increase in tax revenues improves the fiscal stance. Actually, Figure 4 illustrates a positive correlation between the two. The lagged variable captures the persistency of the government expenditure and tax revenues. To be specific, we describe a fiscal policy rule as

(26) $$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 g_t .$$

Using the sample of 1981-2008[11], we obtain the regression result as

(27) $$s_t = -0.021 + 0.658 s_{t-1} + 0.577 g_t , \quad Adj.R^2 = 0.759 .$$

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[10] One percent GDP growth rate roughly corresponds to the "prudent" scenario of the government's mid-term forecast up to 2020 (Cabinet Office, 2010, 2011).

[11] The data of primary balance is available only up to 2008. See Appendix C.

(0.006) (0.160) (0.104)

Based on (27), we set $\gamma_0$ =–0.021, $\gamma_1$ =0.658, and $\gamma_2$ =0.577.

We simulate the model recursively by generating $(g_{t+1}, R_{t+1}, s_{t+1})$ for the stochastic process of $x_{t+1}$, and obtaining $d_{t+1}$, given the starting value of $d_t$.

Table 1 summarizes the parameters that we use for the baseline calibration. The simulation procedure is described in Appendix D.

## 5. Simulation Results

### A. Baseline Forecasts

Table 2 shows the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio exceeds $d_{2009}$ (=1.792)[12]. Under the baseline parameters, the average interest rate is 1.8 percent and the average growth rate is 1.0 percent, with the gap of 0.8 percentage points. The average primary surplus is -4.5 percent of GDP. The expected debt-to-GDP ratio reaches 11.8 in 100 years and continues to increase afterwards, suggesting that debt is not sustainable.

The path of the expected debt-to-GDP ratio and the probability that the debt-to-GDP ratio will exceed its initial value are highly sensitive to the intermediation cost. If we set $\kappa = 0.0$, the average interest rate goes up to 3.33 percent, and the GDP

[12] We chose year 2009 as *t*=0. So, *t*=20 (“20 years after”), for example, indicates year 2029 in Tables 2 to 4 and Figure 6 below.

growth rate to 2.34 percent, respectively[13]. The gap between the two rates goes up from 0.8 to 0.99 percent, and the expected debt-to-GDP ratio increases more rapidly than in the benchmark case, reaching 13.6 in 100 years (the second row of Table 2). The smaller intermediation cost makes fiscal sustainability more difficult, since the estimated elasticity of intertemporal substitution of consumption is below two.

The path of the expected debt-to-GDP ratio is sensitive also to the elasticity of intertemporal substitution of consumption. If we set the inverse of the elasticity of intertemporal substitution at one (the log-utility case) by keeping the average real GDP growth rate to 1.0 percent as in the benchmark case, the average real interest rate goes up to 2.26 percent, making fiscal sustainability more difficult to meet: the expected debt-to-GDP ratio reaches 15.2 in 100 years.

Incorporating the financial intermediation cost into the model ends up with imposing a looser condition on sustainability, given that the elasticity of substitution on consumption is less than two. Recent studies report the elasticity of intertemporal substitution between 1 and 2.[14] We may safely judge that Japanese fiscal policy is not

[13] In Section 5A (Tables 2), we adjust $\gamma_0$ so that the average primary surplus is the same as in the benchmark case (-4.5 percent of GDP).

[14] Abe and Yamada (2005) provide unique micro data evidence of Japanese households, obtaining the estimated elasticity between 1.50 and 1.78. The other evidences of Japan are based on aggregate data. Hamori (1996) and Hatano and Yamada (2007) obtained the estimated elasticity between 1 and 2 in most of their specifications. Sugo and Ueda (2008) and IIboshi et al. (2006) obtained even lower values, which are below 1. Okubo (2003) obtained somewhat mixed results, obtaining the estimated elasticity between 1 and 2 in half of his four

sustainable if the expected debt-to-GDP ratio increases unboundedly based on our model with the financial intermediation cost.

### B. Alternative GDP Growth Rates

We have thus far assumed that the long-run average real GDP growth rate is 1 percent. Here we examine the effects of higher real GDP growth rates on sustainability to see if the high growth helps restore sustainability. Specifically, we adjust the average return on capital, $x^a$, to attain the 2 percent and 3 percent average GDP growth rates. The other parameters are set as in the baseline case. The 2 percent case corresponds to the “Growth Strategy Scenario” in the government’s mid-term forecast up to 2020 (in Cabinet Office, 2010, 2011; Cabinet Decision, 2010). It is also close to the average real GDP growth rate over the period of 1981-2009 (2.2 percent). The second and third rows of Table 3 show that the debt-to-GDP ratio grows more slowly as the average growth rate goes up: in the 1 percent case (the baseline case) the debt-to-GDP ratio reaches 11.8 in 100 years, while the corresponding figures in the 2 percent and 3 percent cases are 6.7 and 3.3, respectively. A higher growth contributes to reduce the future debt-to-GDP ratio through two channels. First, it increases the primary surplus under the estimated fiscal policy rule. The average primary surpluses are -2.8 percent of GDP in the 2 percent case and -1.1 percent in the 3 percent case, respectively. Secondly, as the growth rate goes up, the average real interest rate also rises, but the gap declines, resulting in a lower expected debt-to-GDP ratio. The average interest rate in the 2

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specifications and above 2 in the other half specifications. As far as we know, Fuse (2004) is the only study that consistently obtained the estimated elasticity above 2 percent case is 2.37 percent, and the gap is 0.37 percentage points. In the 3 percent case, the average interest rate is 2.94 percent, and the gap becomes negative, -0.06. It is notable, however, that even if the economy grows at 3 percent, debt is not sustainable given the current fiscal policy rule.

### C. Alternative Fiscal Policy Rules

The above results show that government debt is not sustainable under a wide range of parameters given the current fiscal policy rule. In this subsection, we consider alternative fiscal policy rules. The parameters are the same as in the baseline case except for the fiscal policy rule.

First, we examine whether or not the government's fiscal reconstruction target is sufficient to restore sustainability. The government released that it aims at turning from the current primary deficit of the central and local governments to a primary surplus by fiscal year 2020 (Cabinet Decision, 2010) and estimated that it must increase primary balance by 4.2 percent of GDP to achieve this goal in their "prudent" scenario (Cabinet Office, 2011). Taking this target into consideration, we assume that the primary balance as a proportion of GDP linearly increases and reaches zero in 2020 and that the average primary balance is zero after 2020. The first row of Table 4A reports the simulation result, showing that the expected debt-to-GDP ratio reaches 5.2 in 100 years and continues to increase afterwards. Importantly, just realizing zero or slightly positive primary balance is not sufficient to restore fiscal sustainability.

One simple way to restore sustainability would be to raise the average primary surplus by raising the value of $\gamma_0$ in the rule (26). We find that the primary surplus that is 1.96 percent of GDP on average is enough to stabilize the expected debt-to-GDP ratio at its initial value and, thus, to make debt sustainable[15].

A more flexible and maybe more interesting way to restore sustainability is to change the fiscal policy rule. As Bohn (1998) addresses, a rational government should increase the primary surplus when the debt-to-GDP ratio is high. We incorporate his idea into the fiscal rule by assuming that the primary surplus-to-GDP ratio depends on the lagged debt-to-GDP ratio as well. To be specific, we have

$$s_t = \gamma_0 + \gamma_1 s_{t-1} + \gamma_2 g_t + \gamma_3 d_{t-1}. \tag{28}$$

We use the same parameters for $\gamma_1 (= 0.658)$ and $\gamma_2 (= 0.577)$ as in the baseline case, and set $\gamma_3$ arbitrarily.[16] To compare with the benchmark case, we adjust $\gamma_0$ so that the average primary surplus is –4.5 percent of GDP as in the baseline case if $d_{t-1}$ is at its value as of 2010.

The second to fifth rows of Tables 4A report the expected debt-to-GDP ratios and Table 4B report the associated expected primary surplus-to-GDP ratios for several values of $\gamma_3$. We find that a sufficiently large positive response to the debt-to-GDP

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[15] In Discussion, we conduct the simple calculation concerning how much consumption tax rate will be necessary if the primary surplus of 2 percent of GDP is to be achieved only by the increase by the consumption tax rate.

[16] To avoid possible endogeneity, we estimated (28) using $d_{t-2}$ as an instrument and found that the coefficients on $d_{t-1}$ was positive but not significant. Figure 5 illustrates no significant positive relationship between $s_t$ and $d_{t-1}$, though it does not control for the GDP growth rate. Bohn (1995) estimates the determinants of the US primary surplus, finding that the coefficient of the lagged debt-to-GDP ratio is around 0.03 to 0.05, depending on the sample period (Table 1).

ratio stabilizes the debt-to-GDP ratio and thus makes government debt sustainable. If $\gamma_3$=0.001, the debt-to-GDP ratio increases unboundedly. On the other hand, if $\gamma_3$=0.01, it eventually stabilizes, though at a high level of 4.9. If $\gamma_3$=0.03 and 0.05, the debt-to-GDP ratios stabilize at about 2.7 and 2.3, respectively. As $\gamma_3$ goes up, the required primary surplus becomes initially higher but eventually lower. Raising $\gamma_3$ results in a reduction in the long-run debt-to-GDP ratio, which requires less interest payment and hence less primary surplus. As $\gamma_3$ goes up, from 0.01 to 0.03 and 0.05, the long-run expected primary surplus decreases from 4.5 percent to 2.5 percent and 2.2 percent in terms of GDP, respectively. Figure 6 depicts the expected primary surplus-to-GDP ratio in the case of $\gamma_3$=0.05. It shows that if the debt-to-GDP ratio is to stabilize at 2.3 in the long run, the current primary deficit should turn to a surplus in 10 years, increase up to 2.2 percent surplus of GDP in 20 years, and keep that level afterwards. Comparing to the case under the past rule (in the baseline simulation), the primary surplus-to-GDP ratios under Bohn's rule with $\gamma_3$=0.05 is higher by 4.8 percentage points in 10 years and by 6.6 percentage points in 20 years and later. To follow this Bohn's rule, the government must decrease expenditure and/or increase taxes gradually for the coming 20 years. Step-by-step increases in consumption tax rates may help (at least partly) achieve this goal (see Discussion below). If the debt-to-GDP ratio is to stabilize at a level lower than 2.3, the primary surplus must increase more rapidly. A delayed fiscal consolidation will result in a higher debt-to-GDP ratio and make sustainability more difficult to be restored.

## 6. Conclusion

We investigate fiscal sustainability of Japan by providing a dynamic stochastic general equilibrium (DSGE) model that features low interest rates of government bonds relative to the economic growth rate. Although we have proposed a framework that explains lower intersect rates and provides looser conditions for sustainability than the standard one, our simulation suggests that the government debt is not sustainable if the fiscal policy rule taken over the last thirty years goes over in the future. We can safely say that Japanese government debt is not sustainable unless the fiscal policy rule is changed.

The usefulness of our approach depends on the estimated elasticity of substitution on consumption. If it is over 2, our approach incorporating the intermediation cost ends up with providing a tighter condition for sustainability than the standard model otherwise. In that case, the standard neoclassical growth model may predict that the fiscal policy is sustainable even if our model suggests that it is not. Fortunately, however, the estimates in Japan and the US are both sufficiently less than 2 (e.g., Sakuragawa, et.al. 2010). We have to be careful to apply this approach to other countries.

One useful extension is to incorporate the default cost into the model. The default cost would make the sustainability conditions more difficult to be met because the interest rate would be higher and the debt-to-GDP ratio would increase more rapidly.

### Discussion: Consumption Tax Rate Required to Restore Sustainability

If fiscal sustainability is to be restored only by raising consumption tax rate, how much increase in tax rate is needed?

Eq. (27) implies that the long-run primary deficit ($\frac{\gamma_0 + \gamma_1 g}{1-\gamma_2}$) is 4.5percent of GDP if the average GDP growth rate is 1 percent ($g = 0.01$). We can regard this long-run deficit as structural (i.e., cyclicality- and momentum- adjusted) one. On the other hand, to achieve fiscal sustainability under the 1 percent average GDP growth rate, the primary surplus must be 2.0 percent of GDP. The difference between the structural and targeted primary balance (6.5 percentage points) is a primary surplus that is necessary to be raised either by a decrease in expenditures or an increase in revenues.

The consumption tax revenue as a proportion of nominal GDP is 2.0 percent on average during the period of 1997-2009, when the tax rate was 5 percent. If we do not take into consideration possible effects of the consumption tax on consumption and economic growth, we can suppose that raising consumption tax by one percentage point contributes to a 0.4 percentage increase in primary surplus-to-GDP ratio. Under this assumption, we can mechanically compute that consumption tax rate must be increased by 16 percentage points, i.e., from 5 to 21 percent, to raise primary surplus-to-GDP ratio by 6.5 percentage points.

If the government is to fill in the gap of primary surplus-to-GDP ratios between Bohn's rule with $\gamma_3$=0.05 and the past rule (in the baseline simulation) just by raising consumption tax rates, it has to raise the consumption tax rate by 12 percentage points in 10 years and by 16.4 percentage points in 20 years from the current 5 percent level.

Note that we have to take into consideration negative effects of consumption tax rates on consumption or increases in social security expenditures associated with population aging in order to conduct more accurate estimates.

**Appendix A: Proof of Proposition**

Suppose that $E_t \lim_{s\to\infty} \frac{D_{t+s}}{Y_{t+s}} = \infty$. Then, for some realizations of states, $\lim_{s\to\infty} \frac{D_{t+s}}{Y_{t+s}} = \infty$. Below we limit our attention to those exploding paths. The growing debt-to-GDP ratio should imply that the public debt will crowd out private lending and at some date (denoted by $T$), the credit market will disappear. Agents start financing investment only by their net worth. The NPG condition of agents as of time $T$ is then given by

(NPG2) $$E_T \lim_{s\to\infty} \frac{D_{T+s}}{(1+R_{T+1})(1+R_{T+2})\ldots(1+R_{T+s})\ldots} = 0.$$

Letting $g_t^D$ denote the growth rate of debt, (NPG2) is rewrriten as $E_T \lim_{s\to\infty}\left(\frac{1+g_{T+s}^D}{1+R_{T+s}}\right) < 1$,

which, in turn, implies that there is some $\delta > 0$, satisfying

(A1) $$\lim_{s\to\infty}(R_{T+s} - g_{T+s}^D) > \delta$$

for some realizations of states. In addition, given that $x_t$ is bounded above, $R_t$ is bounded above from the relationship between $x_t$ and $R_t$. Therefore, (A1) implies that $g_t^D$ is bounded above. Let $\bar{g}^D$ denote the upper bound of $g_t^D$. We can rewrite (FC) as

(FC') $$\frac{T_t}{Y_t} = \frac{R_t - g_t^D}{1+g_t^D}\frac{D_t}{Y_t} + z < \tau$$

for all period $t$ and all the realizations of states. For $R_t > g_t^D$, (FC') can also be rewritten as $\frac{D_t}{Y_t} < \frac{1+g_t^D}{R_t - g_t^D}(\tau - z)$. The RHS is bounded by $\frac{1+\bar{g}^D}{\delta}(\tau - z)$, which contradicts that $\frac{D_{t+s}}{Y_{t+s}}$ grows unboundedly. Q.E.D.

## Appendix B. General Solution for Growth and Interest Rates

We derive the equilibrium growth and interest rates. We first solve for $\{g_1, g_2,,, g_n\}$. Letting $G_\alpha$ denote the $n\times 1$ vector with its $j$ th element of $(1+g_j)^{-\alpha}$, $\Phi$ denote the $n\times n$ matrix with the $(i,j)$ element of $\phi_{i,j}$, $\tilde{X}$ denote the $n\times n$ diagonal matrix with the $(j,j)$ element of $1+x_j$, and $I$ denote the $n\times 1$ unit vector, we can rewrite (16) as $I=\beta(1+\kappa)^{-\frac{1}{2}}\Phi\tilde{X}G_\alpha$, which leads to

(A2) $$G_\alpha=\beta^{-1}(1+\kappa)^{\frac{1}{2}}\tilde{X}^{-1}\Phi^{-1}I\cdot$$

Next we solve for $\{r_1, r_2,,, r_n\}$. Letting P denote the $n\times 1$ vector with its $jth$ element of $(1+r_j)$ and $\tilde{G}_\alpha$ denote the $n\times n$ diagonal matrix with the $(j,j)$ element of $(1+g_j)^{-\alpha}$, we can rewrite (17) as $I=\beta\theta^\alpha\Phi\tilde{G}_\alpha P$, which leads to

(A3) $$P=\beta^{-1}\theta^{-\alpha}\tilde{G}_\alpha^{-1}\Phi^{-1}I=\beta^{-1}(1+\kappa)^{-\frac{1}{2}}\tilde{G}_\alpha^{-1}\Phi^{-1}I,$$

where the second equality comes from (14). Letting $X$ denote the $n\times 1$ vector with its $j$ th element of $1+x_j$, we can rewrite (16) as $I=\beta(1+\kappa)^{-\frac{1}{2}}\Phi\tilde{G}_\alpha X$. Substituting this into (A3), we obtain

(A4) $$P=(1+\kappa)^{-1}X.$$

Finally, we compute the steady-state values of $g_t$ and $r_t$. Let the vector $\lambda$ denote the stationary distribution of $\Phi$. Then the steady state values of $g_t$ and $r_t$ are obtained by $g=\sum_{j=1}^{n}g_j\lambda_j$ and $r=\sum_{j=1}^{n}r_j\lambda_j$, respectively, where $\lambda_j$ is the $j$ th element of $\lambda$.

## Appendix C. Data

1. Primary balance is obtained from the current and capital accounts of the system of national accounts (*93SNA*, Economic and Social Research Institute) of the general government as follows:

Primary balance = (Taxes on products and imports + Current taxes on income and wealth + Social burdens + Other current transfers received + Fixed capital depreciation + Capital transfer received) **–** (Subsidy + Social benefit except for social transfers in kind + Other current transfers paid + Final consumption + Gross fixed capital formation + Increases in inventories + Net purchase of land + Capital transfers paid).

The primary balance data based on *93 SNA* is available only up to 2008. To estimate the primary balance in 2009 and 2010, we used the government's estimate that the primary balances of the central and local governments are -8.1 percent and -6.4 percent of nominal GDP in fiscal years 2009 and 2010, respectively (Cabinet Office, *Midterm Economic and Fiscal Forecasts*, June 22, 2010). To convert the government's estimate to the primary balances of the general government (i.e., the total of the central government, local governments, and the social security funds), we assumed that the primary balance of the social security funds as a proportion of nominal GDP as of fiscal year 2008 (-1.1 percent of nominal GDP) did not change constant up to 2010.

2. Nominal and real GDP are based on *93 SNA*, which is obtained from the website of Economic and Social Research Institute.

Nominal and real GDP data are available up to 2009. For the real GDP growth rate in 2010, we used the government's estimate for fiscal year 2010 (2.6 percent, Cabinet Office, 2010).

3. Interest rate margin is the difference between the deposit rate and the lending rate, both of which are obtained from IMF's *International Financial Statistics*.

4. Real yield on financial bills and government bonds are nominal yields on each asset minus the change in GDP deflator. Those data are obtained from IMF's *International Financial Statistics*.

5. Government debt is total debt minus financial bills outstanding. Data source of government debt is *Flow of Funds*, obtained from the website of Bank of Japan.

**Appendix D: Simulation Procedure**

1. Construction of the initial value of $d$

In the benchmark case and the other cases except for *Fiscal Management Strategy* case, we conduct the stochastic simulation from year 2011. To do so, we need to construct the initial value of $d_{2010}$. To do so, we first construct $R_{2010}$ by assuming perfect foresight and substituting $g_{2010}$ into the deterministic version of (14),

(A5) $$\log(1+R_{2010}) = \alpha \log(1+g_{2010}) - \frac{1}{2}\log(1+\kappa) - \log\beta .$$

Then, we substitute $R_{2010}$, $g_{2010}$, $d_{2009}$ and $s_{2010}$ into (22) to get $d_{2010}$ (=1.860).

In the case of *Fiscal Management Strategy*, we conduct the stochastic simulation from year 2021. For year 2010, we follow the same procedure as the benchmark case above. For the period from 2011 to 2020, by assuming that $g_t$ =0.02 (constant) and that $s_t$ linearly increases from $s_{2000}$ (=-0.075) to $s_{2020}$ =0 , we construct $d_{2020}$ using the similar formulae of (A5) recursively.

2. Stochastic Simulation

Step 1. We generate a series of $x_{t+1}$ for 1000 periods starting from the initial value drawn from the stationary distribution of $x_t$ .

Step 2. Given a series of $x_{t+1}$, we obtain $g_{t+1}$ and $R_{t+1}$ from (A2) and (A4).

Step 3. Given a series of $g_{t+1}$, we construct $s_{t+1}$ recursively from the fiscal policy rule (26) with a starting value of $s_{2000}$.

Step 4. We construct $d_{t+1}$ by substituting $R_{t+1}$, $g_{t+1}$, and $s_{t+1}$ into (22) with a starting value of $d_t$ as of year 2010.

Step 5. We repeat Steps 1–4 $N$ times to obtain the distribution of $d_{t+1}$. Indexing each series by $i$, the expected value of $d_t$ and the probabilities that $d_t$ exceeds its critical values $\bar{d}$ are computed as $E[d_t] = \frac{1}{N}\sum_{i=1}^{N} d_{i,t}$ and $\Pr ob\left[d_t \geq \bar{d}\right] = \frac{1}{N}\sum_{i=1}^{N} I\left(d_{i,t} \geq \bar{d}\right)$, where $\bar{d} = 1.792$, $N = 10000$ , and $I\left(d_{i,t} \geq \bar{d}\right) = \begin{cases} 1 & if \quad d_{i,t} \geq \bar{d} \\ 0 & otherwise \end{cases}$ .

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Table 1. Paramters

| | | |
|---|---|---|
| Preference | | |
| $\beta$ | discount factor | 0.980 |
| $\alpha$ | inverse of elasticity of intertemporal substitution | 0.565 |
| Technology | | |
| $x^a$ | average return to capital | 0.033 |
| $\rho$ | serial correlation of return to capital | 0.780 |
| $\sigma$ | standard deviation of error term in return to capital | 0.011 |
| Financial Intermediation | | |
| $\kappa$ | financial intermediation cost | 0.015 |
| Fiscal Policy Rule | | |
| $\gamma_0$ | Constant | −0.021 |
| $\gamma_1$ | coefficient on previous-year primary surplus/GDP | 0.658 |
| $\gamma_2$ | coefficient on GDP growth rate | 0.577 |

Table 2. Expected debt-to-GDP ratio under alternative parameters

| After | 20 years | 50 years | 100 years | 500 years | 1000 years |
|---|---|---|---|---|---|
| baseline | 3.21 | 5.76 | 11.84 | 582.79 | 49118.59 |
| | (93.9%) | (98.1%) | (99.7%) | (100.0%) | (100.0%) |
| no intermediation cost | 3.32 | 6.23 | 13.62 | 1209.97 | 229444.07 |
| | (94.9%) | (98.7%) | (99.8%) | (100.0%) | (100.0%) |
| log utility | 3.41 | 6.53 | 15.15 | 2597.90 | 1242884.42 |
| | (99.7%) | (100.0%) | (100.0%) | (100.0%) | (100.0%) |

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.792)

Table 3. Expected debt-to-GDP ratio under alternative growth rates

| After | 20 years | 50 years | 100 years | 500 years | 1000 years |
|---|---|---|---|---|---|
| GDP Growth=1% (baseline) | 3.21 | 5.76 | 11.84 | 582.79 | 49118.59 |
| | (93.9%) | (98.1%) | (99.7%) | (100.0%) | (100.0%) |
| GDP Growth=2% | 2.69 | 4.00 | 6.74 | 74.97 | 799.67 |
| | (84.7%) | (89.5%) | (95.1%) | (100.0%) | (100.0%) |
| GDP Growth=3% | 2.23 | 2.64 | 3.31 | 9.07 | 17.60 |
| | (68.5%) | (67.7%) | (70.7%) | (83.7%) | (90.2%) |

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.792)

Table 4. Alternative fiscal policy rules

A. Expected debt-to-GDP ratio

| After | 20 years | 50 years | 100 years | 500 years | 1000 years |
|---|---|---|---|---|---|
| Average primary surplus/GDP=0 | 2.47 | 3.31 | 5.20 | 190.24 | 15854.42 |
| | (91.8%) | (81.1%) | (81.8%) | (89.1%) | (89.8%) |
| Bohn's rule $\gamma_3$ = 0.001 | 3.18 | 5.50 | 10.42 | 200.42 | 4154.41 |
| | (93.9%) | (98.1%) | (99.7%) | (100.0%) | (100.0%) |
| Bohn's rule $\gamma_3$ = 0.01 | 2.94 | 3.94 | 4.62 | 4.96 | 4.91 |
| | (94.0%) | (97.8%) | (99.0%) | (99.4%) | (99.3%) |
| Bohn's rule $\gamma_3$ = 0.03 | 2.56 | 2.65 | 2.66 | 2.67 | 2.65 |
| | (93.3%) | (94.2%) | (94.4%) | (94.5%) | (94.2%) |
| Bohn's rule $\gamma_3$ = 0.05 | 2.33 | 2.32 | 2.31 | 2.32 | 2.32 |
| | (92.1%) | (91.0%) | (91.1%) | (91.3%) | (91.4%) |

1. Numbers in the parentheses are the probabilities that the debt-to-GDP ratio exceeds its value as of year 2009 (1.792)

B. Expected Primary Surplus-to-GDP Ratio

| After | 20 years | 50 years | 100 years | 500 years | 1000 years |
|---|---|---|---|---|---|
| Bohn's rule $\gamma_3$ = 0.001 | -4.03% | -3.48% | -2.00% | 52.63% | 1188.60% |
| Bohn's rule $\gamma_3$ = 0.01 | -1.75% | 1.34% | 3.47% | 4.60% | 4.46% |
| Bohn's rule $\gamma_3$ = 0.03 | 1.24% | 2.43% | 2.48% | 2.53% | 2.48% |
| Bohn's rule $\gamma_3$ = 0.05 | 2.16% | 2.04% | 2.13% | 2.10% | 2.15% |

**Figure 1**

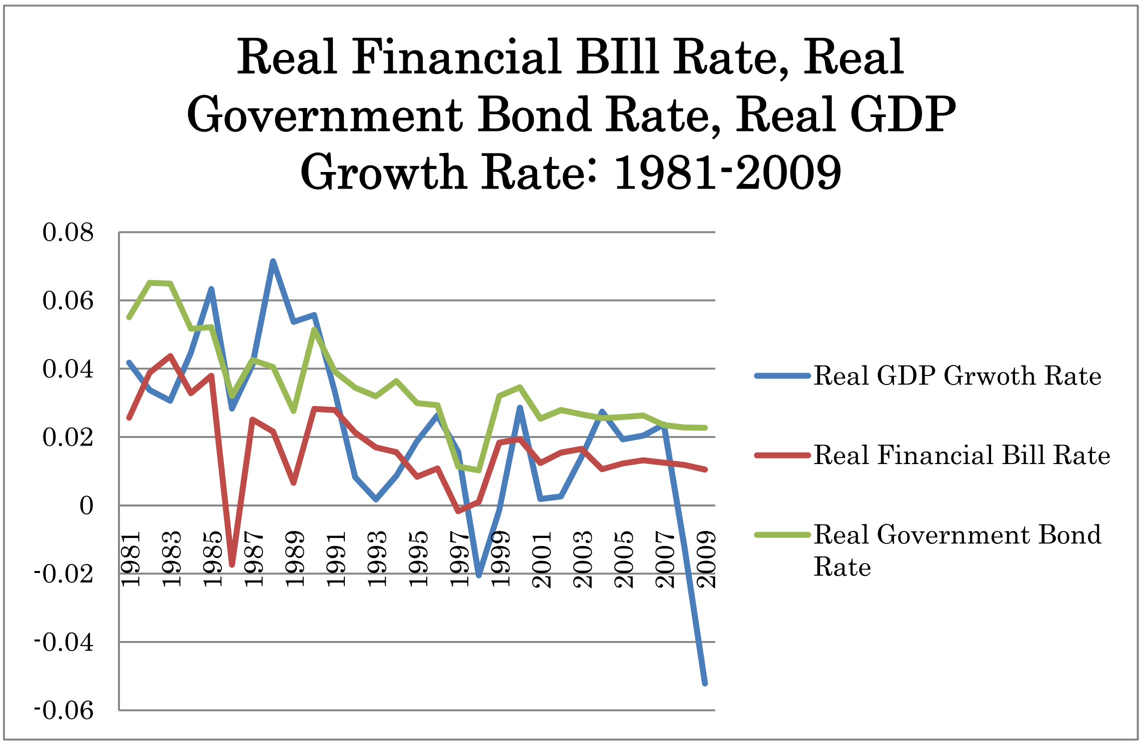
Real Financial BIll Rate, Real Government Bond Rate, Real GDP Growth Rate: 1981-2009
0.08
0.06
0.04
0.02
0
-0.02
-0.04
-0.06
1981
1983
1985
1987
1989
1991
1993
1995
1997
1999
2001
2003
2005
2007
2009
Real GDP Grwoth Rate
Real Financial Bill Rate
Real Government Bond Rate

**Figure 2A.**

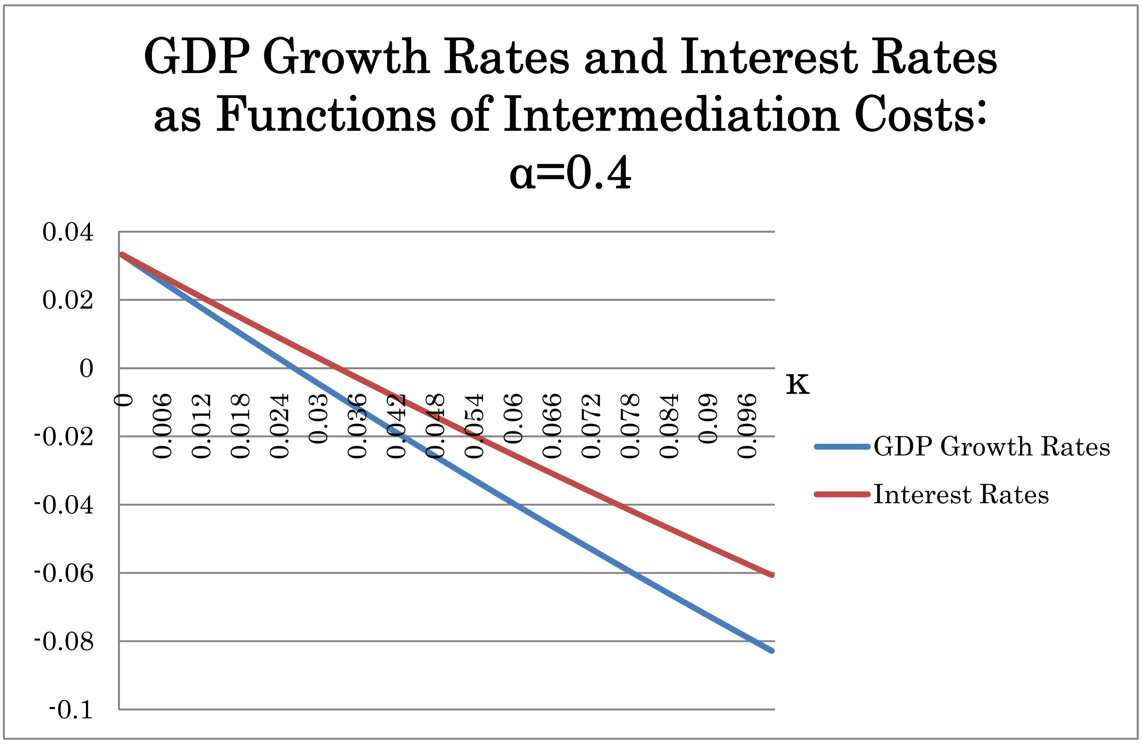


**Figure 2B.**

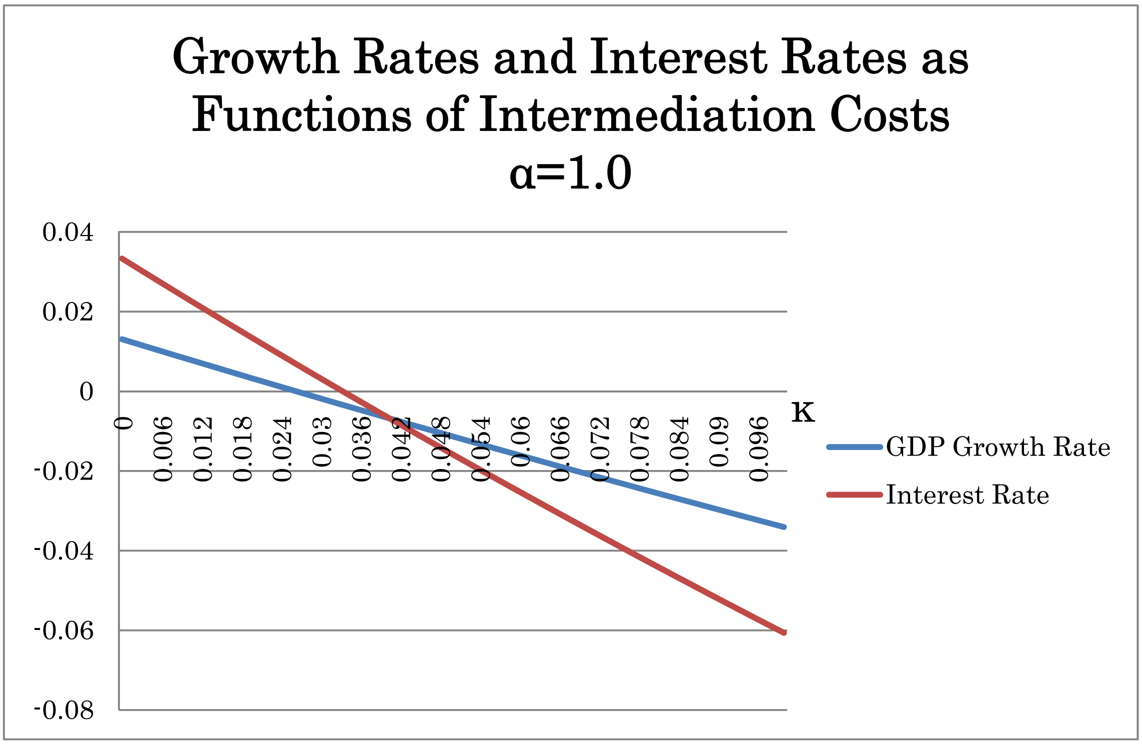

**Figure 3**

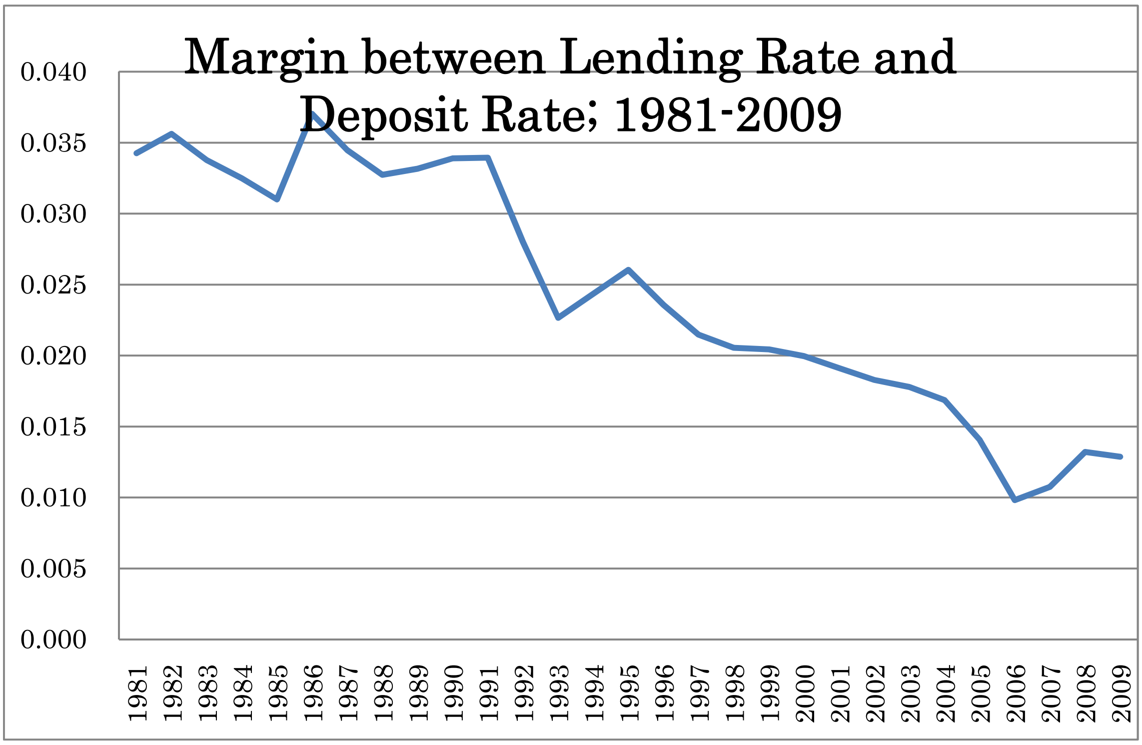


**Figure 4**

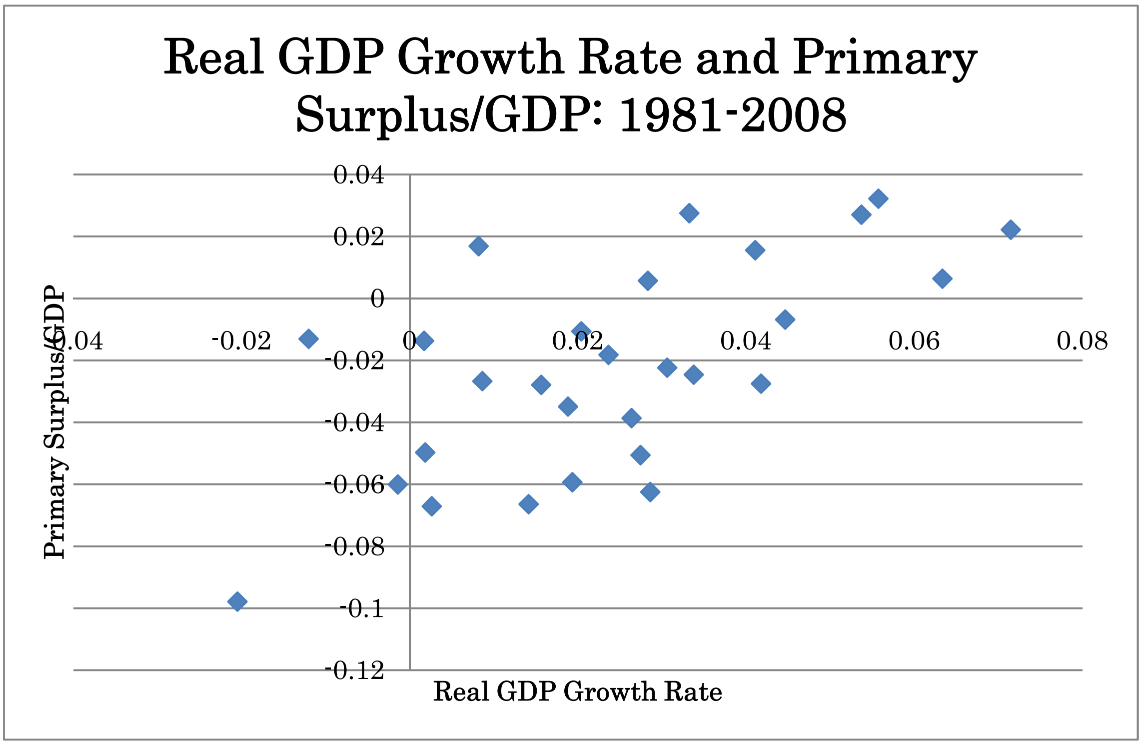

**Figure 5**

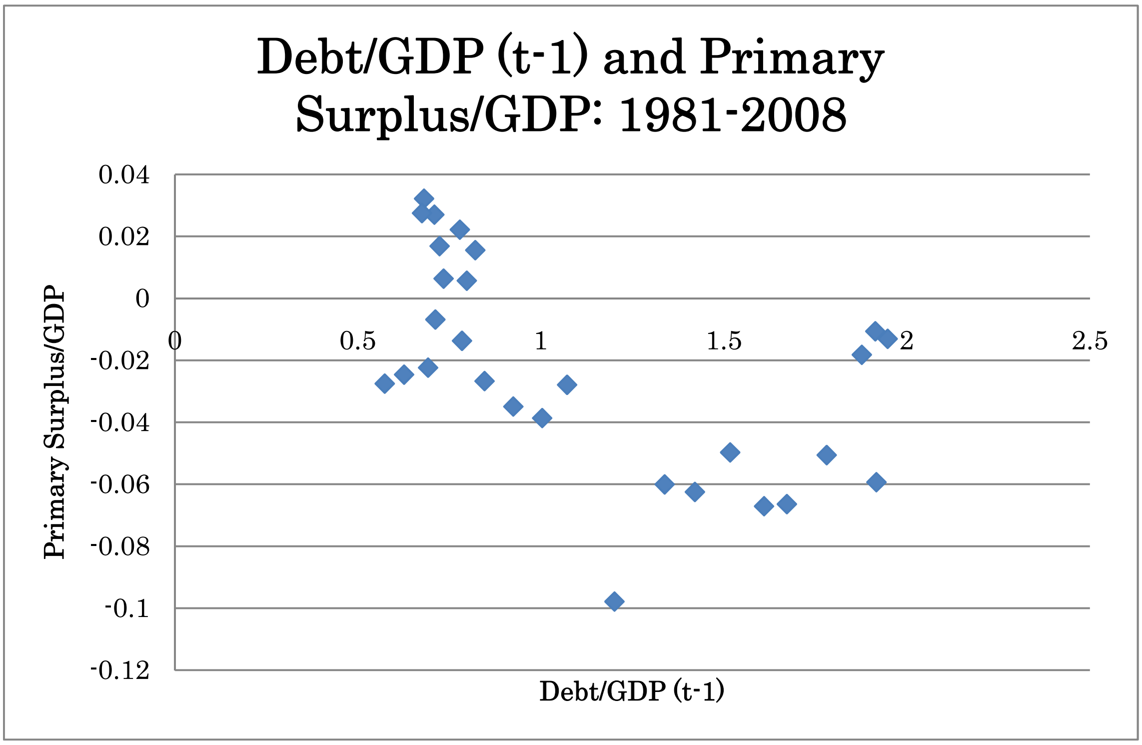


**Figure 6**